# Math 31 - Homework 1

Due Friday, June 29

## Easy

**1.** Do Section 1.5, #1. [Note: The author uses $(a, b)$ to mean $\gcd(a, b)$.]

**2.** Verify that the following elements of $\langle \mathbb{Z}_n, \cdot \rangle$ are invertible, and find their multiplicative inverses.

(a) 4 in $\mathbb{Z}_{15}$

(b) 14 in $\mathbb{Z}_{19}$

**3.** In each case, determine whether $*$ defines a binary operation on the given set. If not, give reason(s) why $*$ fails to be a binary operation.

(a) $*$ defined on $\mathbb{Z}^+$ by $a * b = a - b$.

(b) $*$ defined on $\mathbb{Z}^+$ by $a * b = a^b$.

(c) $*$ defined on $\mathbb{Z}$ by $a * b = a/b$.

(d) $*$ defined on $\mathbb{R}$ by $a * b = c$, where $c$ is at least 5 more than $a + b$.

**4.** Determine whether the binary operation $*$ is associative, and state whether it is commutative or not.

(a) $*$ defined on $\mathbb{Z}$ by $a * b = a - b$.

(b) $*$ defined on $\mathbb{Q}$ by $a * b = ab + 1$.

(c) $*$ defined on $\mathbb{Z}^+$ by $a * b = a^b$.

**5.** Do Section 2.1, #1 of Herstein.

## Medium

**6.** Suppose that $*$ is an associative and commutative binary operation on a set $S$. Show that the subset

$$H = \{a \in S : a * a = a\}$$

of $S$ is closed under $*$. (The elements of $H$ are called **idempotents** for $*$.)

**7.** Do Section 2.1, #9.

**8.** Do Section 2.1 #13. We handled groups of order 1, 2, and 3 in class, so you just need to prove that any group of order 4 must be abelian. [Hint: There will be two possible group tables in this case.]

**Extra credit:** Try to extend Problem 8 by showing that any group of order 5 must also be abelian. [This is exercise #21 of Section 2.1 in Herstein.]